# Speculations on the Origins of Linear Perspective 

Article in Nexus Network Journal • April 2003
DOI: 10.1007/s00004-002-0005-5

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Newcastle University
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# Richard Talbot $\mid$ Speculations on the Origins of Linear Perspective 

Including analyses of Masaccio's Trinity and Piero's Flagellation

Richard Talbot demonstrates an approach and method for constructing perspectival space that may account for many of the distinguishing spatial and compositional features of key Renaissance paintings. The aim of the paper is also to show that this approach would not necessarily require, as a prerequisite, any understanding of the geometric basis and definitions of linear perspective as established by Alberti. The author discusses paintings in which the spatial/geometric structure has often defied conventional reconstruction when the strict logic of linear perspective is applied.

## Introduction

The purpose of this paper is to demonstrate an approach and method for constructing perspectival space that may account for many of the distinguishing spatial and compositional features of key Renaissance paintings. The aim of the paper is also to show that this approach would not necessarily require, as a prerequisite, any understanding of the geometric basis and definitions of linear perspective as established by Alberti. In particular, the paper discusses paintings in which the spatial/geometric structure has often defied conventional reconstruction when the strict logic of linear perspective is applied. It specifically examines the spatial construction of four very different paintings in order to explain how the geometry and methods involved may shed light on Brunelleschi's architecture, as well as on some of the questions and issues surrounding the history, origins and nature of linear perspective.

Briefly, I propose that an explanation for the unique compositions and the apparent inconsistencies of many paintings is that their spatial structures have not been generated purely using the logic of linear perspective. I would argue that their distinctive characteristics are not the result of making a projection from a ground plan or constructing a pavimento. Rather they are the result of developing a space using a particular two-dimensional geometric construction-a matrix-to create a ready-made framework into which the imagery is then fitted. Further I would say that the imagery within the paintings is sometimes directly inspired by the geometry and imagery of the matrix itself. The matrix contains both a surface grid/pattern and the diminishing proportions that provide the characteristic convergence, and the controlled changes of scale necessary to create the spatial illusion. I will demonstrate that this kind of matrix can be developed simply from the geometric patterns found in pre-Renaissance paintings.

## Background

The evidence and the ideas that I present here are the direct result of research that involved re-thinking the geometric constructions of two key Renaissance paintings. These are Masaccio's Trinity, (circa 1427), which is thought to be the earliest painting to show a systematic approach to spatial diminution, and Piero della Francesca's Flagellation. ${ }^{1}$ These paintings are considered to be particularly significant because of the artists' respective links with Brunelleschi, credited with discovering perspective, and Alberti, credited with the first written account of the theory of linear perspective in 1435. The importance of the Flagellation, (circa 1455?), relates not only to the apparently rigorous application of linear perspective, but also to its distinctive and innovative composition and to Piero's own status as a perspective theoretician. Martin Kemp has also commented on a property of the compositions of both paintings that is of particular relevance and importance for my thesis. ${ }^{2}$ He has noted that the artists appear to show complete control over the conjunctions of elements that are spatially unconnected within the paintings. Elements that are on the surface of the paintings or in shallow space appear to coincide with, or complement in some way, elements that are deeper in space. As I will demonstrate, I believe that this phenomenon is a direct result of the type of geometric construction used by the artists, namely a matrix.

My research has also been fuelled by reading James Elkins's The Poetics of Perspective [1994], and his article "The Case Against Surface Geometry" [1991], in which he is critical of attempts to analyse early Renaissance paintings in terms of their surface geometry and perspective. In this article, he raises important general questions about the analysis of paintings and examines the relationship, if any, between surface geometry, perceived harmonious relationships, and perspective constructions-surface geometry defined as being systematic non-illusionistic geometry. He also discusses Wittkower's [1953] interpretation of Brunelleschi's use of perspective in relation to proportion in his buildings. I feel broadly sympathetic to James Elkins's sentiments about the nature of perspective and his reservations regarding reconstructions, and I have attempted to pay heed to his criticisms in my own approach to these paintings. My approach is also informed by my own experience as an artist who uses linear perspective and for whom the matrix of lines implicit in a perspective construction itself acts as a vehicle or medium for the imagination. As a student, I learned the mechanics of perspective directly from Piero della Francesca's De prospectiva pingendi and was intrigued, not only by the nature of perspective projection, but equally fascinated by the potential spatial ambiguity of perspective diagrams, simple geometric forms and patterns.

I will now set out the broad areas that are particularly relevant for my thesis, and then show how they are relevant to the construction of certain paintings. I will look at Alberti's construction, the orthodox history of perspective and some of its assumptions, and the various geometric patterns found in paintings, the significance of which I believe has been overlooked.

## A Brief Description of the Orthodox History of Perspective

Linear perspective, the geometry of which appears to have its origins in the early fifteenth century, has come to be thought of mainly as a tool for painters, enabling them to represent spatial relationships systematically on a two-dimensional surface. ${ }^{3}$ As far as geometry is concerned, it is the projection of three-dimensional spatial relationships from a single point, onto a surface.


Fig. 1.
For a painter, this surface is usually, but not necessarily flat, and is known as the 'picture plane'. The discovery of perspective is attributed to the architect Brunelleschi, and it has been suggested that it originated in his desire to understand the mechanism that governs the apparent diminution of architectural elements according to their position and distance from the eye. ${ }^{4}$ This knowledge would then have enabled him to control the relationship between the real space within a building and the projected image of that space. The subsequent adoption of linear perspective as a tool by artists/painters appears to have its roots in a general desire to represent or depict the third dimension more accurately or convincingly. The assumption is that artists wanted to solve the problem of creating a more logical, measurable, naturalistic and unified space and that Brunelleschi's discovery provided the solution. However, the precise nature of that solution and the mechanism and date of its discovery remain unclear. ${ }^{5}$ Reliefs by Donatello are the earliest works to show a relatively consistent approach to diminution in space, but Masaccio's Trinity, (circa1427), on which it is believed Brunelleschi collaborated, is held to be the first painting to fully utilize the new knowledge of perspective and to show systematic and accurate proportional diminution. ${ }^{6}$ It is therefore generally assumed that the method used in the spatial construction of Masaccio's Trinity reflects, or in some way relates to Brunelleschi's original findings.

In 1435, some eight years after Masaccio's Trinity was painted, Alberti described the general principles for creating a regularly diminishing floor grid and provided the first theoretical account of what we now call linear perspective (Fig. 2). ${ }^{7}$


Fig. 2.
He described it as a projection and explained the relationship between the position of the eye, the cone of vision and its intersection with the picture plane. He described two separate constructions, the information from which is combined to give a receding perspectival grid. Consequently, most analyses of the perspective constructions of Renaissance paintings post 1435 , are based on the tacit assumption that the artists would be using Alberti's method in one form or another, or would be aware of the principles underlying Alberti's theory. ${ }^{8}$ It is assumed that the artist would initially have created a perspectival grid on the ground (pavimento), or would have made projections directly from an architectural ground plan, having first made a conscious decision regarding the eye level and distance from the picture plane. The receding grid on the floor would then act as a guide to the relative scale of all other elements within the picture. Alberti suggests relating the viewer's height, (3 braccia), to the size of the floor squares, ( 1 braccia), and also states that a check for correctness is the convergence of the diagonals of the squares on the ground to a single point.

In reality, however, there are paintings that show an approach that could not be considered to be purely Albertian. Many paintings show a floor grid with a recession that appears to be governed solely by the $45^{\circ}$ diagonals of the grid squares being drawn towards a point at eye level, often placed at the edge of the painting. This approach is often referred to as the 'distance point' method and these points are known as 'distance points' simply because the distance between them and the central vanishing point is the same as the distance between the viewer and the picture plane. ${ }^{9}$ It follows that if the vanishing point for the orthogonals is placed centrally, and the edge of the painting is used as a distance point, then the 'correct' viewing distance is half the width of the painting. It also follows that the angle of view is $90^{\circ}$.

It has been generally assumed that these points have been placed at the edge of the paintings for completely practical reasons. However, Alberti's description of the mechanics of the perspective construction is, in fact, slightly ambiguous and open to various interpretations. Samuel Y. Edgerton demonstrated that Alberti's description could imply that the viewing point should be placed at the edge of the painting, and that the artist subsequently decides the position of the picture plane [Edgerton 1975: 40-49]. The two diagrams described by Alberti are, in effect, overlaid, and under certain
circumstances, when the picture plane is placed down the centre of the painting, would result in the distance point and the viewer's position coinciding. The particular properties and wider implications of this construction, however, appear to have gone unnoticed.

The following two figures show two of many possibilities. In Fig. 3 the viewer's distance from the picture plane is half the width of the picture, making the angle of view $90^{\circ}$. The resulting transversals are found to be placed at $1 / 3$ and $1 / 2$ the height of the rectangle.


Fig. 3.
In Fig. 4, the viewer is $1 / 4$ the width of the picture from the picture plane, and the resulting transversals are found to be placed at $1 / 2,2 / 3$, and $3 / 4$ the height of the rectangle.


Fig. 4.

These forms of the Albertian construction, where the viewer, the picture plane and grid units along the base are in a defined relationship to each other, are particularly important. Under normal circumstances regular surface grids and perspective diminution do not mix, but in this particular case they do. The resulting divisions within the rectangles follow simple ratios, creating simple harmonic grids based on the reciprocals of whole numbers. It is known that both Piero della Francesca and Leonardo da Vinci were interested in and researched these specific relationships [Wittkower 1953].

For these relationships to hold, the distance from the eye to the picture plane must be the same as the length of the grid units along the base (Fig. 5).


Fig. 5.

Fig. 6 shows another way of developing the same divisions, and Fig. 7 shows that the transversals of a perspectival grid drawn within such a rectangle divide along their length into $4,5,6,7$ etc, equal parts.


Fig. 6.


Fig. 7.
Alberti's text can be interpreted as describing a general principle of projection, with no predetermined measured relationship between the plan, the picture plane and the viewer. The construction that results lends itself to the drawing of a simple rectangular space, based on a rectangular floor subdivided into squares.

Edgerton's interpretation is more specific and results in a construction which contains very particular geometric and number properties. Additional orthogonals are easily inserted where the transversals touch the sides of the rectangle, resulting in a space that is not confined by two dominant orthogonals from the two lower corners of the rectangle (see Fig. 7). The construction fills the whole rectangle and is consequently more evocative of larger and more open spaces, a quality that can be seen in the under-drawing of Uccello's Nativity (ca. 1450, Soprintendenza alle Gallerie, Florence).

Equally important is the fact that it is a construction that can be developed without any regard for the concepts of 'projection' and the 'picture plane'. I suspect, for reasons that will become apparent, that it is this type of construction and the simple harmonic relationships within it that are the key to Brunelleschi's architecture and his involvement with the development of the geometry of perspective.

## Floor Patterns

The general view is that the use of geometric linear perspective by painters in the early fifteenth century is the natural culmination and inevitable conclusion of developments and changes initiated by earlier painters such as Giotto, Duccio and Pietro Lorenzetti. ${ }^{10}$ It is thought that because these painters were apparently attempting to deal with space, using overtly perspectival elements such as tiled floors, chequered cloths, coffered ceilings and 'vanishing' points, they were undergoing a major shift in sensibility and were moving consciously, or otherwise, towards a more 'naturalistic' representational method. An increased knowledge and understanding of optics and other practical forms of geometry such as surveying, map making and astronomy, together with Brunelleschi's investigations into controlled diminution within architecture lead to the so-called
discovery of linear perspective. The elaborate floor patterns within Renaissance paintings could be seen as an example of the kind of challenge that could now be undertaken using the new science of linear perspective.

It has been said that the very elaborate floor present in Piero's Flagellation is one of the most complex patterns to be found in any early Renaissance painting [Wittkower and Carter 1953]. Domenico Veneziano's St. Lucy Altarpiece (ca. 1445, 209 cm x 216 cm, Gallerie degli Uffizi, Florence) also contains an elaborate tiled floor that appears to be unique. However, both patterns can, in fact, be seen in various forms in much earlier paintings. The $\sqrt{ } 2$ pattern of the Flagellation is clearly visible in paintings by Gaddi and Jacopo di Cione. ${ }^{11}$ The pattern within the floor of the St. Lucy Altarpiece, which is usually interpreted as being derived from a hexagon, is also present within the six-pointed star pattern in the floor of Ambrogio Lorenzetti's Presentation of Christ at the Temple, painted nearly one hundred years before. Because of its apparent complexity and the presence of sixty degree angles, it has been assumed that the floor in the St Lucy Altarpiece could only be achieved through a method of perspectival projection but that does not account for Lorenzetti's use of the same pattern. ${ }^{12}$

The explanation for this is that, despite appearances, the St. Lucy Altarpiece floor is not a true hexagonal pattern, but is one that can be derived from the repeated subdivision of squares (Fig. 8). I think that it would be fair to assume that Domenico Veneziano was following a process essentially the same as that used by Lorenzetti (Fig. 9).


Fig. 8.


Fig. 9. The floor of Domenico Veneziano's St. Lucy Altarpiece


Fig. 10.


Fig. 11.

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It has been thought that long before Brunelleschi's breakthrough there was a 'rule of thumb' workshop practice which enabled painters such as Lorenzetti to draw basic receding tiled floors. I believe, however, that the existence of this particular Lorenzetti floor showing a faux hexagon demonstrates a level of geometric sophistication and understanding that goes well beyond a workshop 'rule of thumb'. It shows an involvement in a piece of geometric play that, as I will shortly demonstrate, when taken a few stages further, produces the geometry of Albertian perspective. I would argue, therefore, that the use and manipulation of these patterns may be a significant step in the development of the geometry behind linear perspective.

Stepping back fifty years before Lorenzetti, Giotto uses the same basic patterns extensively within the border patterns of many paintings (Figs. 10 and 11). They can also be seen in the Baptistery in Florence and in Roman Herculaneum. These are patterns that look complex but that can all be derived from the simple subdivision and overlapping of squares. The main process is one of placing one square diagonally within another, a process that produces diminishing proportions 2:1.414 :: 1:0.707, and so on.

Figs. 12, 13, and 14 show the process of the successive subdivision of a square. This produces a spatial construction that corresponds to an Albertian perspective construction as interpreted by Edgerton. Edgerton's model uses points on the picture's sides as either viewing points or distance points, and therefore contains harmonic proportions which correspond to those that appear in the subdivision of a square.

It is the nature of this construction that it can be extended in any direction and all the relationships still hold true (in Masaccio's Trinity, this construction has been extended upwards). It is a construction that contains a regular surface grid but is, at the same time, spatial. The heights of the transversal up the picture plane follow the simple harmonic sequence, $1 / 2,1 / 3,1 / 4,1 / 5$, etc., and the successive transversals are naturally divisible by orthogonals into 2, 3, 4, 5, etc. (see Fig. 7). It also follows that the divisions on a transversal that is divided by orthogonals into five equal parts will correspond vertically to the divisions on a transversal divided into ten. The divisions on a line divided by orthogonals into four will correspond to the divisions on a line divided into eight, and so on. It is also a construction that allows easy further subdivisions of the floor grid, giving the impression that a distance point beyond the edges of the painting has been used. This can be achieved because every individual square of the receding floor has diagonals running through it, enabling it to be further divided by a new transversal. Thus, what was ostensibly a square, can become two squares, the new diagonals of which would meet at a distance point beyond the edges of the painting. This is essential for a construction such as that in the floor of Domenico Veneziano's St. Lucy Altarpiece. (Although I have shown this construction based on a square, the construction works for any rectangle).


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Figs. 15, 16 and 17 show the construction extended sideways.


Fig. 15.


Fig. 16.


Fig. 17.

Fig. 17 shows the base line divided into thirds and the height of the viewer, as suggested by Alberti, corresponding to three of the units on the base line. Most important of all, it can be seen that the basic principles of perspective, as described by Alberti and shown in Fig. 18, are contained within this construction.


Fig. 18.
It is conceivable, therefore, that the geometry of linear perspective may have been born out of a simple piece of flat geometry derived from patterns, and not from a conscious desire to understand and rationalise space.

## The Paintings and their Construction

As I mentioned above, I would suggest that an explanation for both the distinctive compositions and the apparent inconsistencies found within many paintings is that the spatial structure has not been generated purely using the logic of linear perspective. The paintings are, I believe, the result of developing a space using the kind of matrix that I am presenting here. The matrix provides a readymade framework into which the imagery can be fitted; it not only generates the diminishing proportions that control depth within a painting, but can also be used to govern the proportions of elements on its surface.

The matrix construction is a 'surface construction', but the web of lines within it can be used both to represent a three-dimensional spatial structure and to regulate the surface design. Elements become interchangeable, so that spatial elements can become surface elements, and vice versa. The ambiguous space of Leonardo's Last Supper (1497), which has defied reconstruction, may be the result of such an approach.

The interplay between elements that are in relatively shallow space and those that are in deep space, accounts, I believe, for the many visual alignments and coincidences seen in the work of Masaccio and Domenico Veneziano. Most important of all, the alignments and control over the positions of spatially unconnected elements, as noted by Martin Kemp, are a natural characteristic of this approach and would be impossible to achieve had the paintings been constructed entirely using a logical Albertian perspective projection. ${ }^{13}$ The concept of the picture plane, the height of the eye and its distance from the picture plane, plans and elevations, etc., can all be effectively ignored.

I am giving examples of paintings where I believe the imagery within the paintings is directly inspired by the geometry and imagery of the matrix itself. I wish to emphasise the use of the matrix because of its combined geometric and visual/spatial properties. It becomes an integral part of the imagery and allows the composition, linear spatial structure and other spatial devices such as colour and tone, to be simultaneously developed. I believe that it has been a crucial part of what was originally an exciting, playful, and imaginative creative process. It enabled artists to combine an intuitive response to the creation of space with a rational, technical response.

## Masaccio's Trinity

If one stands centrally, at a suitable distance from Masaccio's Trinity and at an average person's height, the illusion created is of real figures in a real architectural space that extends beyond the surface of the wall on which it is painted. ${ }^{14}$ It appears to be a space that has been constructed following the logic of linear perspective.

There are many aspects of Masaccio's Trinity, however, that have lead me to question the way in which the space in this painting might have been constructed. ${ }^{15}$ Some of the more obvious inconsistencies, discrepancies and peculiarities are described below.

On close inspection, some of the details of the architecture and the space are clearly fudged, and are not what would be expected from an accurate or semi accurate projection from a plan. ${ }^{16}$

The construction of the Ionic columns has been carried out between two vertical lines, and the method behind the construction of the ellipses at the top shows that they have not been projected. The correct way of drawing the column, if it had followed a grid or the logic of a projection, would have been to construct a three-dimensional box in which the column is then drawn. The top and bottom of the box would facilitate the accurate drawing of the ellipses and this would then help in the modelling of the column's surface.

The connection between the orthogonal moulding at the base of the coffered barrel vaulting and the vaulting itself does not make three-dimensional sense. There are conflicting visual clues to how the vaulting springs from the moulding. The painting of the underside of the moulding suggests that it has width, but the depiction of the meeting between the vaulting, the far rear vertical plane and the horizontal plane of the moulding, implies that it has no width.

The spacing of the lower coffers and ribs within the ceiling is irregular.
It is not possible to establish a consistent and sustainable viewing distance for all parts of the painting. The $45^{\circ}$ diagonals of all the square abaci should, in theory, meet at the same point (a so-called distance point) on the horizon. They do not.

Many elements in the painting seem to coincide visually with other elements that are spatially unrelated to them, that is, something near coincides or is aligned with
something unconnected further away. For instance, the nails in Christ's hands, the lowest rib in the ceiling, and the volutes on the Ionic capitals are all aligned with the 'vanishing point'. The moulding orthogonal to the picture plane from which springs the barrel vaulting, and the upper, outer corners of both the Ionic capitals and the Corinthian capitals are all aligned. The outer edges of each pair of short columns below the donors' ledge coincide on the painting's surface with the edges of the pilasters above. Because the paired columns are closer to the spectator than the pilasters, this coincidence is unexpected.

Another aspect of the painting, which is linked to the previous one, is that of the proportions and construction of the architectural elevation as it sits on the picture surface and then as it recedes in space. Why are key points within the architecture aligned, both within the elevation on the surface and also in receding space? The orthogonal moulding and the outer corners of the Ionic capitals are understandably aligned to the vanishing point, but this alignment also appears then to determine the width of the pilaster.

These characteristics, which are unlikely to be accidental, are indicative of an approach that enables the composition and spatial structure to be simultaneously developed, the various elements being fitted into a predetermined area on the picture surface. These alignments and the control over the positions of spatially unconnected elements would be impossible to achieve had the painting been constructed using a perspective projection.

Masaccio has instead used a flat geometric surface construction that mimics perspective, one that contains diminishing proportions and in which there is convergence to a point-the 'vanishing point'. ${ }^{17}$ Because the mouldings at the base of the barrel vault converge at an angle very close to $36^{\circ}$, the possible explanation could be that he used a construction that involved a pentagon. However the angle $36^{\circ} 52^{\prime}$ is easily derived from a construction based on squares. The converging angle within the matrix is twice the angle whose tangent is $1 / 3$ (see Fig. 19).

Figs. 19, 20, 21, and 22 show the development of the painting. It can be seen that the front pilasters in the painting have been formed within a receding plane.

Much of the lower part of the painting, that is, the tomb section, is badly damaged and so what is visible now is largely a reconstruction by restorers based on fragments.


Fig. 19.


Fig. 20.


Fig. 21.

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Fig. 22.

## Uccello's Hunt In A Forest

The panel on which Paolo Uccello's Hunt in a Forest (ca. 1467, Ashmolean Museum, Oxford) is painted is 177 cm wide $\times 73.3 \mathrm{~cm}$ high, giving it the precise proportions of $2.414: 1$, or $(1+\sqrt{ } 2): 1$. The painted area is slightly less wide and high than this. In order to demonstrate the general principle behind the construction, I am only showing the lower part.

I propose that both the imagery and the spatial construction within this painting are directly derived from a matrix (Fig. 23). It has been suggested that in order to counteract the problems associated with a wide panorama, Uccello created three independent perspective boxes, placed side by side, thereby disrupting the logic of a single central vanishing point. ${ }^{18}$ This idea has been prompted because there are several areas of focus along the length of the painting. I think, however, it is clear from the drawing that these apparent additional areas of focus are a consequence of being able to make more connections within a grid other than just by means of the $45^{\circ}$ diagonals.


Fig. 23.
Although my construction is more elaborately drawn, this construction concurs with the findings and with the perspective construction proposed by Martin Kemp and Ann Massing following the physical examination of the painting during conservation at the Hamilton Kerr Institute at the University of Cambridge [Kemp and Massing et al. 1991]. The painting shows evidence of a minimal pavimento, but Kemp and Massing also observed that at any given depth within the scene the height of a horse plus rider corresponds with, and therefore is probably based upon the distance between the main orthogonals at that particular depth. This, I would argue, combined with the fact that all the riders' eye levels are on the horizon, is a natural consequence of the distance between the main orthogonals at the picture plane being equal to the height of the horizon, which is itself a consequence of the surface divisions being based upon the square (Fig. 24). The heights of the trees could also then be established by a similar method, their heights being based on multiples of a chosen unit.


Fig. 24.
I am also tempted to suggest that the composition of Uccello's Profanation of the Host (ca. 1468, Galleria Nazionale delle Marche, Urbino), where there also appear to be several viewpoints along its length, may follow a similar procedure as that of the Hunt. ${ }^{19}$ In the under-drawing of Uccello's Nativity (formerly in S. Martino alla Scala, Florence), it can similarly be seen that he was not only using the edges of the painting as distance points, but he was also using the points where the lower transversals touch the edge of the painting as starting points for new orthogonals. ${ }^{20}$ Although the Nativity drawing is based on a rectangle rather than a square, this is a property only of harmonic constructions, where the distance point is placed at the edge of the painting.

## Leonardo's Last Supper

The many analyses of Leonardo's Last Supper (ca. 1497, S. Maria della Grazie, Milan) have all noted the ambiguities and the conflicting clues about the real nature of the space. Whatever assumption is made regarding the shape or size of a specific part, there is conflicting evidence from elsewhere, making it impossible to establish a single viewing position that satisfies every aspect of the painting. It is as if the walls, the floor, the ceiling and the table have all been constructed and treated separately.


Fig. 25.

I propose a simple explanation for these problems. Leonardo has created a perspectival, harmonic grid, into which the composition has been fitted, and has also deliberately mixed spatial and surface elements. He has given priority to simple surface relationships over strict three-dimensional logic, and this has lead to the apparent inconsistencies. For instance, I believe that the front and top surfaces of the table have been formed out of the receding grid on the ground. The top front edge of the table is, therefore, $1 / 2$ the distance between the bottom edge of the painting and the horizon, and the bottom edge of the table is $1 / 4$ the height. The height of the back edge of the ceiling is $1 / 3$ the distance between the top of the painting and the horizon. He has taken the two sections either side of the central section of ceiling and has made them part of the walls.

## Piero's Flagellation

Piero della Francesca was in a position, as his writings testify, to construct the space in his paintings methodically by using the theory of perspective as described by Alberti [della Francesca 1942]. He could project from a plan, not only the architecture, but if need be, the figures also. However, my analysis of the geometry and perspective construction of Piero's Flagellation (ca. 1455, Galleria Nazionale delle Marche, Urbino) shows that his overall approach is, in some respects, similar to that which I believe was used by Masaccio in the Trinity. He has linked elements of the receding architecture to a spatially suggestive two-dimensional surface matrix, the geometry of which also provides key points for the perspective construction. ${ }^{21}$


Fig. 26.
The pattern in Fig. 26, derived from an octagon, is the basis of the elaborate floor pattern within the palace where Christ is standing. It was used in earlier paintings by Gaddi and Cione, and contains the $\sqrt{ } 2$ proportions that control the composition of the painting.


Fig. 27.
Piero della Francesca has initially developed a surface pattern based on the physical $\sqrt{ } 2$ proportions of the painting (see Fig. 27), which is also the basis of the $\sqrt{ } 2$ pattern found within the elaborately tiled floor. He then takes the idea a step further by using key points in that surface pattern as the central vanishing point and the distance points (Fig. 28). 22 It demonstrates how Piero creates not only spatial, but also conceptual links within the composition. He does this not by using perspective purely as a device for creating an illusion, but by employing the matrix of constructional lines and the format of a geometric perspective construction as part of the creative and imaginative, imageforming process.


Fig. 28.

He has constructed transversals placed at harmonic intervals $1 / 2,1 / 3$, and $1 / 4$, and has then used the centre of the 'mother' square as the position of Christ's head. Christ stands on the $1 / 3$ transversal at a point that now becomes the centre of the square formed using the $1 / 2$ and $1 / 4$ transversals (Fig. 28). This nominal square is then subdivided creating the three squares inside the Palace as well as the associated white banding (Fig. 29). Christ, therefore, has been placed centrally in squares both in a vertical plane and the horizontal plane.


Fig. 29.
This perspective sleight of hand has enabled Piero to create a deeper space than could be achieved by using the edges of the painting as distance points. He has drawn a square that is then further divided by transversals. The distance points of the newly created squares would be well beyond the edges of the painting.

The central square contains the circle on which Christ stands, and the other two, in front and behind, contain the elaborate black and white tiling. The pattern within the tiling is achieved by simply projecting part of the $\sqrt{ } 2$ surface geometry of the painting into the floor plane.


Fig. 30.

The proposed constructions reveal a direct and unambiguous geometric link between Christ and the central figure standing in the foreground, whose stance reflects that of Christ's (Fig. 30). Both figures are standing on the same diagonal, and a $22.5^{\circ}$ line from the left-hand distance point grazes Christ's head and almost touches the young man's eye. He could be looking at, remembering, or imagining the scene. Other significant aspects of the painting are revealed through the construction. For example, the nearer flagellator's right arm and whip are in a square at the exact centre of the painting. The two-dimensional $\sqrt{ } 2$ star pattern and its radiating lines are directly associated with the architrave of the two doors. The $45^{\circ}$ diagonal from the top left hand corner of the painting, connects to the central vanishing point, defines part of the ceiling construction and the edge of the white banding on the floor. The surface pattern also creates a system of radiating lines that could be considered to be 'vanishing' to, and emanating from Christ's head (Fig. 31).


Fig. 31.
Piero has successfully integrated a logical and coherent system of linear perspective with a spatially suggestive, two-dimensional geometric pattern. Changes of scale across the surface geometry mimic the spatial effects generated by the perspective construction and, although Piero has had the advantage of a complete understanding of the geometry of linear perspective, he has, nonetheless, chosen a route that, like Masaccio's Trinity, allows him to relate elements of the receding architecture to elements within the surface geometry. Similarly, if Piero had constructed the painting following Alberti's method
and had projected from a plan, it would have been virtually impossible for him to predict or control the positions of elements on the picture surface.

The analysis and reconstruction of this painting also supports and verifies the observation made by James Elkins regarding the apparent similarity between the composition of Piero's Flagellation and some of Piero's diagrams in De prospectiva pingendi [Elkins 1994: 107; Elkins 1987]. This similarity is a natural consequence of the distance point and viewing position coinciding within a perspective construction that is also a harmonic construction (Fig. 31). A harmonic construction in this context is a rectangle in which successive horizontal divisions are placed at intervals based on the reciprocals of whole numbers, and this will occur within a perspective construction when the distance points lie at the edges of the painting. The resulting transversals are found to be placed at $1 / 2,2 / 3,3 / 4$ the height between the base line and the eye level. Looking at the construction as an elevation, the left hand distance point could be considered to be the viewing position, the centre vertical division of the painting could be seen as the picture plane, and the bottom right hand corner as the front edge of the Palace. The same relationships exist if those elements are then looked at in plan. In effect, the surface pattern within the rectangle of the painting contains all the elements required for a perspective construction, that is, the plan, the elevation, the picture plane and the position and height of the viewer.

## Brunelleschi

As a consequence of the ideas that I have discussed above, I have been prompted to speculate about the original nature of Brunelleschi's discovery, the earliest dating of which is 1413 [Kemp 1992: 9-14]. I ask the following:

- If my proposed approach to generating perspectival space has played a role in the construction of the paintings I have described, how do these geometric constructions relate to both Brunelleschi's discovery of the geometry of perspective and to his architecture (specifically, San Lorenzo, the Ospedale degli Innocenti, ca.1419, and Santo Spirito, ca.1434)?
- Could it be that Brunelleschi was already aware of these two-dimensional harmonic spatial constructions and his investigations were related to the construction of the real space that these images might represent?
- If Brunelleschi was Masaccio's collaborator on the Trinity, ca. 1427, can this painting be considered to be putting Brunelleschi's investigations into practice, not in three-dimensions, but on a flat surface? ${ }^{23}$

Brunelleschi may have been attempting to solve a problem when he discovered the geometry of perspective, but it could be argued that it may not have been the problem, associated now with painting, of depicting a convincing space on a picture surface. ${ }^{24}$ As I have demonstrated earlier, these perspectival grids can be constructed without any reference to or knowledge of the concept of the picture plane. The issue of the picture
plane might only arise once there is an attempt to build a real space corresponding to the image. An attempt that might have involved the building of a space corresponding to an existing perspectival harmonic grid.

As Alberti later showed in De pictura, 1435, linear perspective is a process of projecting three-dimensional spatial information from a single point onto a twodimensional surface. It follows that this mechanism can also work in reverse and that a two-dimensional image can be projected into a three-dimensional space. ${ }^{25} \mathrm{~A}$ space could be designed and constructed in such a way that, when viewed from a particular position, it would present a predetermined two-dimensional image to the viewer. ${ }^{26} \mathrm{~A}$ similar phenomenon is experienced in the instance of looking up with one eye into a uniformly lit ribbed dome, where one sees simultaneously a flat radiating pattern and a real threedimensional structure.

For Brunelleschi, this investigation could be achieved by using models, the twodimensional image being projected from a single point into the real three dimensional space, either using the eye or strings. The original image/matrix, would have been a linear geometric construction, but because, for example, the real pillars have a dimension, this would have to have been taken into account within the grid of the matrix. This, in effect, would create a module that corresponded to the units within the matrix and to which all the other dimensions in the building would then be related. ${ }^{27} \mathrm{He}$ would then be able to create a real space in which receding elements would appear to coincide with elements from the two-dimensional surface. The space could be stretched or compressed, the scale and position of the various elements being determined according to their position and distance from the eye. Unexpected alignments and placements could be made, but they could continue to be faithful to the original image. It is out of this experimentation that I believe an understanding of the underlying geometry of projection, the concept of the 'picture plane', the importance of the position of the eye and hence linear perspective would have arisen naturally.

## Brunelleschi's Painted Panels

None of what I am proposing may provide any immediate solution to the nature of the two small painted panels used by Brunelleschi to demonstrate perspective and which are described by his biographer Manetti. It is these panels, particularly the one depicting the Baptistery in Florence, that have attracted the most attention in the debate surrounding the development or invention of perspective. ${ }^{28}$

The many questions include:

- What were these panels actually demonstrating? The presumption must be that Brunelleschi was painting the Baptistery in Florence because it was an octagonal building, and was therefore suitable for demonstrating the apparent convergence of its $45^{\circ}$ sides to equidistant points on a horizontal line at eye level. I think that this phenomenon would only be evident if those distance points were to lie physically on the painting. After all, an octagonal building
had already been painted very successfully by Duccio one hundred years earlier (Christ's Temptation in the Temple, ca. 1310, Museo dell'Opera, Siena), so for Brunelleschi to have simply achieved this again would not have been a breakthrough.
- What was the viewing angle? It has been said that it could have been between $53^{\circ}$ or a practicable maximum of $90^{\circ}$. The argument in favour of $53^{\circ}$ is based on it being just large enough to show all that Manetti described as being included within the picture. It also provides a viewing distance equal to the width of the painting. The argument against $90^{\circ}$ is based on it implying a very short viewing distance, to the extent of being uncomfortable (half the width of the panel). The advantage of it being $90^{\circ}$ is that the lateral points of convergence for the $45^{\circ}$ lines, the distance points, would be at the edge of the painting, and would presumably, therefore, enhance the demonstration.
- How were the paintings viewed? Manetti describes looking at the painting of the Baptistery in a mirror via a hole in the back of the painting. The main effect of viewing the picture in this way is that the spatial illusion is greatly enhanced. It makes the viewer scan the picture bit by bit, while being placed centrally and at the correct viewing distance. It also stops the viewer being aware of the actual physical surface of the painting, which would hinder a convincing illusion. I would say, however, that whether the correct viewing angle was $53^{\circ}$ or $90^{\circ}$ would make very little difference to the inherently difficult act of looking at a 30 cm square painting through a small hole. Viewing the picture at $90^{\circ}$ is not much more awkward than at $53^{\circ}$. Additionally, if a $53^{\circ}$ angle was used, then the width of the Baptistery alone takes up much of the panel. If this had been the case, I do not think that Manetti would have mentioned what could be seen on either side of the Baptistery.
- How were the panels made? Manetti gives no indication as to how the images were made, but methods based on contemporary practical knowledge of surveying, astronomy, map making and optics have all been suggested, as well as scaled drawings, painting directly onto a mirror, or using a camera obscura. Martin Kemp has indicated that certain conditions are needed for an invention to occur [1992: 9-14]. One of these is that the end towards which an invention is directed should be considered desirable, which in the case of linear perspective he identifies as the 'systematic recording of visual phenomena'. He also suggests that the images on Brunelleschi's panels were developed from the actual buildings 'working from these towards a perspectival projection', and that 'he was not, therefore, creating an independent space based on a priori principles'.
There is nothing in Manetti's account of these panels, however, that would exclude the images painted on them having been derived from a harmonic grid similar to the
ones I have described above, such as in Fig. 16. In fact, I would suggest that many of the questions and problems associated with understanding the nature of these panels would be solved if the images depicted on them had been made using one of these grids. A harmonic grid, with a suitably low eye level and with distance points placed automatically at the edges of the panel, would have a viewing angle of $90^{\circ}$. This would have enabled Brunelleschi to depict convincingly the Baptistery and its surroundings, and at the same time demonstrate the convergence of $45^{\circ}$ lines to distance points on the painting itself. The same grid could then have been used for the much larger panel depicting the angular view of Palazzo de' Signori in Florence.

Whatever the actual mechanism was, I believe that the proposals in this paper may provide evidence of a parallel area of spatial experimentation which not only Brunelleschi, but many other artists too may have pursued, and may suggest an alternative route or contributing factor in the evolution of linear perspective.

## Notes

1. For Piero's Flagellation see [Wittkower and Carter 1953]; For Masaccio's Trinity see [Field 1997], [Field, Lunardi and Settle 1988], [Aiken 1995], [Kern 1913], [Schlegel 1963], [Janson 1967], [Coolidge 1966], [Polzer 1971], [Sanpaolesi 1962: 42-53] and [Cristiani-Testi 1984].
2. Kemp writes: 'In the Flagellation, the artful ambiguity is less developed, but there is no question that Piero is sharply aware of surface interplays such as those between the sharply silhouetted light and dark forms inside and outside the praetorium. I think it is true in general to say that the greatest perspectivists-we may think of Masaccio, Piero, Leonardo and Saenredam as among such-have not only exhibited complete mastery of the construction of space, but have also shown a heightened awareness of the shapes of forms when projected on to the flat surface of the painting' [Kemp 1992: 32]. Speaking of Domenico Veneziano, Kemp writes: 'His St Lucy Altarpiece not only contains a virtuoso display of advanced perspective but also exploits a marvellously cunning series of visual conjunctions which compress elements at different depths into an interlocked composition' [Kemp 1992: 35].
3. The date for its discovery is suggested by different authors to be anywhere between 1409 and 1425 . It is complicated by the fact that the term 'perspective' was originally linked to the general area of optics, and only later took on its more specific meaning.
4. Wittkower acknowledges that '...he had to invent painters' perspective since the twodimensional projection was the only mathematical way of determining the relation between distance and diminution' [1953: 288]. However, any building containing regular intervals and a fixed height throughout would achieve some of the qualities that Wittkower is claiming for Brunelleschi's buildings, but he goes on to say that 'it was only during the Renaissance that everything was done to make the perception of a harmonically diminishing series a vividly felt experience'. The elevation of a building is parallel to the picture plane and so is unaffected by perspective. There must be something else that distinguishes the image formed by a Brunelleschi interior, from the image formed by any other set of objects placed at regular intervals in space. This is an area that was originally dealt with earlier in Argan [1946].
5. Manetti, thought to be the author of the biography The Life of Brunelleschi, describes Brunelleschi as the inventor of perspective. Manetti claims to have first hand knowledge of the painted panels that were used by Brunelleschi to show perspective It is from Manetti's description that numerous attempts have been made to understand the precise nature of these panels which no longer exist. The main problems are understanding exactly what Brunelleschi did or did not know at the time, what it was he was actually demonstrating, the exact nature of these panels, and how he made them. These problems have generated a multitude of explanations ranging from the panels being made from measured plans and elevations, through to the idea that Brunelleschi was in fact demonstrating paintings made using a camera obscura. See [Manetti 1970]; [Tsuji 1990].
6. The suggested collaboration between Masaccio and Brunelleschi is usually on the grounds of the style of the architecture depicted in the painting.
7. [Alberti 1972]; [Alberti 1966]. The original Latin version, De pictura, was published in 1435. Alberti's description of the mechanics of creating a perspectival floor is brief and incomplete as it was probably not written for artists. The Italian version, Della pittura, the text of which is not a direct translation from the Latin, was published in 1436.
8. A reconstruction of the perspective structure of a painting makes certain assumptions, the most important being that the painting actually contains a rational space. Working backwards, a rational ground plan can then be reclaimed from the painting, and it is often assumed that the artist must have started with such a ground plan. The most well known reconstruction of this type is Wittkower and Carter's analysis of Piero's Flagellation, where the outcome was a ground plan and elevation together with suggestions for the significance of the various geometric and numeric relationships perceived to be in the plan, the elevation and the painting itself. There are several basic problems with this approach, the main one being that linear perspective is a self contained geometric system with its own internal logic and it is possible to fabricate a rational illusory space without knowing or specifying the real nature and dimensions of that space beforehand. It is through the identification of an object of known size and position that the rest of the space becomes measurable, and although a ground plan can usually always be generated, it does not logically follow that the artist necessarily started with a ground plan. It also assumes that the geometric perspective construction is always separate from, and takes priority over, all the other compositional and spatial devices available to the artist.
9. The two vanishing points on the horizon at which diagonal $45^{\circ}$ lines in the horizontal plane meet, are known as distance points. They are the same distance from the central vanishing point as the viewer is from the picture plane. If within a picture, a horizontal square parallel to the picture plane can be identified, extending the diagonals to the horizon will give the distance points. The distance of the viewer to the picture plane is then known, and it becomes possible, by working backwards, to create a plan of the space within the picture. It is debatable whether the correct viewing distance was of any importance to the early users of perspective. It becomes more important when the game playing potential of perspective is realised. There can also be a conflict between this 'ideal' viewing distance, the physical distance that the artist is from the surface while working, and the distance from which the painting is normally seen. For instance, I think that within Piero's Baptism of Christ in the National Gallery, London, Piero may have generated a perspectival grid using a diagonal to the edge of the painting, making the 'correct' viewing distance half the width of the painting. Standing very close to the
painting, one eye shut, level with the horizon, the space within the painting alters dramatically, making sense of the proportions of the figures in the background.
10. This approach to the history of perspective is evident in [Kemp 1992] and [Edgerton 1991].
11. The same pattern that is within the floor can also be seen in at least two paintings in the National Gallery, London. It is in the decoration on the sides of the thrones in The Coronation of the Virgin, 1380-90, attributed to Agnolo Gaddi, and The Coronation of the Virgin, 1370-71, attributed to Jacopo di Cione and Workshop.
12. Kemp [1992: 35-36] suggests that a technique described later by Piero della Francesca in De prospectiva pingedi could have been used to create the correct perspective of the floor.
13. If a picture is generated by following Alberti's theory and method, it is virtually impossible to control the outcome, for example the sizes, positions and angles of all the elements, relative to each other, or in relation to the proportions/edges of the picture plane. The resulting image on the picture plane is a product of various factors, which include the initial decisions about the viewer's height and distance from the picture plane. To manipulate the image intentionally so that it coincides with the 'surface' geometry would be very difficult, if not impossible. Within the Trinity and the Flagellation, there are distinct elements of the receding architecture that are linked to the vanishing point which itself is linked/determined by the 'surface' geometry of the painting.
14. The exact date for the execution of this 6.67 m high fresco in Santa Maria Novella is unknown, but it was probably painted between 1425 and 1428 , and it would have been completed in a time period of weeks rather than months. Because of the nature of fresco painting, the day-to-day stages in its execution are apparent, as are many other indications of the practical problems encountered by Masaccio. The fresco was covered up for almost 300 years by a work by Vasari and on re-discovery in 1861, the fresco was moved to a different part of the church, but without the still covered skeleton's tomb. In 1952, it was moved back and re-united with its recently discovered lower part. Therefore, the painting has only been known in its entirety in modern times for fifty years. Substantial loss has occurred primarily in the area of the lower tomb and the area that is now painted in as steps on which the two lower figures kneel. The original level of the floor is also uncertain.
15. Some of these observations can also be found in [Kemp 1992]; [Field 1997]; [Field, Lunardi, and Settle 1988]; [Kern 1913]; [Schlegel 1963]; [Janson 1967]; [Coolidge 1966]; [Polzer 1971]; [Sanpaolesi 1962: 42-53].
16. The surface marks are visible in photographs made in raking light in [Polzer 1971].
17. This is modern terminology for Alberti's centric point.
18. See [Lloyd and Kleibel 1981]. The section within this exhibition catalogue on the painting's perspective construction was compiled by Sallyann Kleibel and is based on [Sindona 1972].
19. This painting contains six separate but related scenes, the two best known being 'the attempted destruction of the Host' and 'a woman redeeming her cloak at the price of the consecrated Host'.
20. From what little can still be seen of the painting, I would suggest that, having drawn a grid that would normally be interpreted as representing a standard square tiled floor, Uccello simply ignored that interpretation. He then exploited the visual rather than the purely logical aspect of the perspectival grid. This further supports my feeling that for Uccello, these spatially suggestive grids were an aid to the imagination, and not simply representations of rational spaces.
21. There are slight discrepancies within the perspective construction in the painting. I believe that these are probably attributable to Piero having moved the near left hand pillar slightly inwards, so that it would be seen in its entirety.
22. The painting's dimensions are in the region of $58.4 \times 81.4 \mathrm{~cm}$., and I therefore think it is reasonable to suggest that the original intention would have been for the painting to have $\sqrt{ } 2: 1$ proportions. There appears to be a difference in length from the ideal $\sqrt{ } 2: 1$ proportion of up to 10 mm , depending on who does the measuring. This could be seen as a major problem, but I would argue the following:

- Depending on the procedure that is used to create the $\sqrt{2}$ pattern, the discrepancy makes little or no difference. If the construction proceeds with the initial geometric construction of $45^{\circ}$ and $22.5^{\circ}$ angles, any anomaly in the proportions can be virtually ignored. The central vanishing point is achieved immediately, and this then enables the horizon and the two distance points to be placed. As these are key to the $\sqrt{ } 2$ pattern and the perspective, the construction can be based around these points.
- It is difficult to be completely sure of the edges of the painting. The painting was probably done with the frame in place, and its removal has caused unevenness, and loss of small amounts of gesso. I have superimposed the construction at the extreme top edges, but the construction could be started 1 or 2 mm either side of this edge while retaining its general proportions and without destroying the overall good fit.

23. 'Granted that Brunelleschi wanted his buildings to be looked at as if they were projected on to an intersection, the difference between architecture and painting becomes one of artistic medium rather than of kind' [Wittkower 1953: 289-290].
24. It could be that we now wrongly interpret the discovery of perspective as a solution to a problem, as an advancement on previous methods of depiction, and another inevitable step along the road to greater similitude.
25. The mechanism is not symmetrical, as there are an infinite number of possible threedimensional spaces that would correspond to a particular two-dimensional image.
26. This spatial play is precisely the purpose to which the geometry of linear perspective was later put by Bramante, Borromini and Guarini. It is something that can be achieved theoretically, using drawings, projecting the required image in order to generate a set of plans and elevations of the real space. It enabled architects to manipulate the scale and position of elements crucial to the perceived depth of the space, something that Borromini was to use to great effect in Palazzo Spada. It would also allow a painted illusory space to become an integral part of the architecture, as in Bramante's S. Maria presso S. Satiro.
27. I believe that the clearest example of the direct translation of a square based matrix into a building is Michelozzo's Library at San Marco in Florence.
28. For example, see [Kemp 1992: 9-14 and 344-45]; [Willats and Dubery 1983: 58-65]; [Damisch 1994]; [Edgerton 1975: 124-52]; [White 1957: 113-21].

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## About the Author

Richard Talbot is a visual artist who makes large scale drawings involving geometry and perspective. He studied Astronomy and Physics at University College London, and then Fine Art at Goldsmiths' College, University of London, and at Chelsea School of Art. In 1980, he was awarded the Rome Scholarship in Sculpture and spent two years at the British School in Rome during which time he travelled widely throughout Italy and also in Egypt. He continues to exhibit, and currently teaches drawing and sculpture part time at The City and Guilds of London Art School, as well as being a visiting artist at several other art schools and universities in the United Kingdom. In 2002 was awarded a Rootstein Hopkins Foundation grant. His drawings can be seen at http://www.richardtalbot.org.

